

Prediction of tool-chip contact length using a new slip-line solution for orthogonal cutting

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Abstract

Tool-chip contact length is an important parameter in machining. Several ways had been proposed in different works to find its value, which gave discordant results for the same set of cutting conditions. In this paper, a new slip-line solution for orthogonal cutting by tool with whole rake face is suggested. Based on the proposed solution, a new formula for tool-chip contact length has been obtained. Comparative analysis of different methods to predict tool-chip contact length has been done and experimental verification conducted. The suggested formula has shown to correspond well with experimental data and predicts tool-chip contact length better than other known solutions.

Key words: orthogonal cutting, slip-line solution, tool-chip contact length

1. Introduction

Machining of metals is always accompanied by chip formation. The chip contacts with tool rake face during chip movement along the rake face. One of the most important parameters in this process is the tool-chip contact length. Knowing this contact characteristic together with contact stress distribution makes it possible to determine cutting forces and temperature patterns, accurately calculate tool strength, choose the optimal tool geometry, and control the machining process as a whole. Many researches had been devoted to the study of tool-chip contact length [1–9]. The

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general assumption was that two regions of roughly equal length exist in a whole tool-chip contact length. The first is the sticking region close to the tool edge where contact shear stress is usually considered as having uniform distribution and maximum value [7]. This region is also called “plastic” tool-chip contact length since it causes plastic deformation of chip and curling [10]. The second is the sliding region where contact shear stress is assumed to decrease down to zero as power function [7] at the end of tool-chip contact. In this part of the contact, the chip probably acts elastically on the rake face; therefore this length is also called “elastic” tool-chip contact length.

The most important parameter in considering the problem is total tool-chip contact length. This parameter is interrelated with processes in the primary shear zone, the shear angle in particular [1, 2, 5, 6]. In the case of cutting by tool with restricted rake face, tool-chip contact length directly influences shear angle [11]. To predict tool-chip contact length depending on conventional shear angle, many ways have been proposed based on different assumptions. The solution of the given task may depend on suggested slip-line solution [1], assumed stress distribution [2], or additional consideration [6]. In many works, experimental formulas to determine tool-chip contact length have been presented [5, 7–9, 12]. In this paper, a unique analytical way to predict tool-chip contact length is proposed based on new slip-line solution for orthogonal cutting.

2. Consideration of shear stress distribution on tool-chip interface

The theory of plasticity implies that construction of a slip-line field depends on assumed boundary conditions, which, in the case of cutting, are stress distribution on tool rake face. This specially concerns shear stress distribution, which defines the direction of slip-lines on the boundary. According to the various experimental data [7, 13–16] the graphs of this distribution can differ significantly from each other especially when close to the tool tip. Fig. 1 shows three different ideas about shear stress distribution. Such a distinction in experimental results is most probably caused by different ways for stress measurement. Zorev [7] and Bobrov [13] used the split-tool method. However, in these works, whether forces on tool clearance face took into account or not had not been reported. Thus, this study supposes that these forces had been neglected. This neglect, however, leads to serious distortion of the experimental graph of stress distribution, especially in the case of split-tool method application. As shown in Fig. 2, forces N_c and F_c acting on clearance face create additional parasitical stress on contact L_1 because part 1 of split tool measures both forces N_l and F_l on rake face and N_c and F_c on clearance face. These additional error stresses increase close to the tool edge, and this is probably the reason for the shear stress distribution experimentally found by Bobrov (Fig. 1(a)). The graph configuration may be changed depending on cutting conditions and split-tool geometry, including tool rake and clearance angles. For example, it is obvious that decrease of clearance angle leads to increase of forces on clearance face and growth of

error stresses on rake face for the split-tool method. For cutting conditions applied by Zorev [7], these errors probably resulted in shear stress distribution (Fig. 1(b)). It should be noted that the error of stress determination by split-tool application could also be caused by built-up-edge formation and “agglutination” between two parts of the tool.

Gordon [14] changed the design of the split-tool dynamometer, which can decrease the influence of forces on tool clearance face and get the epures of shear stress (Fig. 1(c)). Petruha [15] experimentally found the same view of shear stress distribution for cutting hard-to-machine materials. A similar graph is presented for cutting of lead [5]. Bagchi and Wright [16] applied photo-elastic sapphire tool and got the same behavior of experimental shear stress distribution for steels 1020 and 12L14.

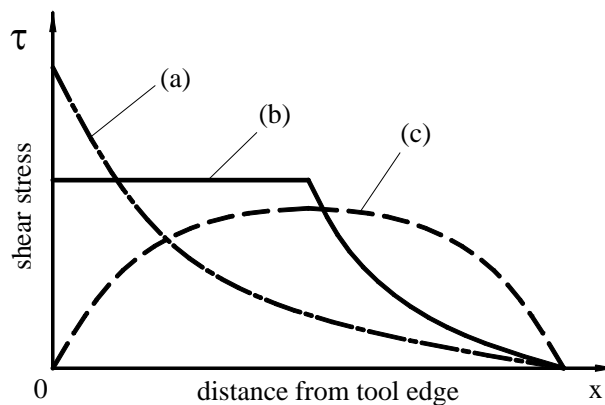


Fig. 1. Principle shear stress distribution on tool rake face according to experimental data of (a) Bobrov [13], (b) Zorev [7], and (c) Gordon, Petruha, and Bagchi and Wright[14–16].

In the last case, the application of photo-elastic sapphire tool makes possible the reduction of errors caused by general split-tool method. At first, the forces on the clearance face would decrease since coefficient of friction is reduced for couple steel and sapphire. By the same reason, the built-up-edge formation is almost impossible in sapphire tool, excluding the errors caused by this phenomenon. The errors of stress determination are distributed more uniformly along the tool-chip contact length and distort the real graph of stress distribution, but not as significantly as the general split-tool method. This implies that experimental shear stress distribution as described in [14–16] (Fig. 1(c)) is more accurate compared with Zorev [7] and Bobrov’s [13] data.

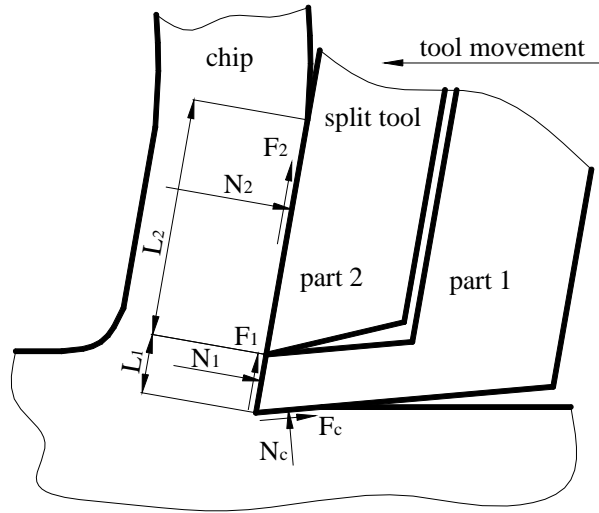


Fig. 2. Scheme of force action in cutting by split tool.

According to the experimental results given in [14–16], the view of shear stress distribution on tool rake face is independent of work material and cutting conditions. In these works, the value of shear stress was found by extrapolation to be always zero at the tool edge. Thus, the sticking effect on “plastic” contact area can be caused by the action of normal stress. Indeed, if shear stress is very small and the normal stress is high close to the tool edge, this means that material particles move along the tool rake very slowly. And according to experimental data [14–16], if the shear stress is zero at the tool edge, then there is no movement of material particles in this area. Material “stops” and “sticks” to the tool rake face forming initial built-up-edge. However, in most cases, its life is very short. The built-up edge itself forms a “new” tool geometry with very high positive rake angle that decreases the forces acting on the “rake face” of the built-up-edge. If the forces on the “clearance face” of the built-up-edge were sufficient, it would be removed from real tool rake face together with tool material particles, and this can be one of the reasons for tool wear. If sticking forces were enough to maintain the built-up-edge on tool rake face, the growth of the built-up-edge would continue until the forces on its “clearance face” exceeded the strength of sticking. Thus, the division of tool-chip contact zone into two parts and their relationship with shear stress distribution, as suggested by Zorev [7], is not quite true. From this point of view, it is more correct to present total tool-chip contact length with assumed stress distribution.

3. Review of some present methods to determine chip-tool contact length

According to Lee and Shaffer’s slip-line solution [1], the length of sticking region on tool-chip interface can be obtained from the formula

$$L_s = a \frac{\sqrt{2}}{2 \sin \Phi \cdot \sin \left(\frac{\pi}{4} + \Phi - \alpha \right)}, \quad (1)$$

where a is the undeformed chip thickness, Φ is the shear angle, and α is the tool rake angle. Using other construction of plastic field inside the chip, Abuladze [6] suggested another formula to define plastic tool-chip contact length:

$$L_s = a \cdot (\xi(1 - \tan \alpha) + \sec \alpha), \quad (2)$$

where $\xi = \frac{a_1}{a}$ is the chip thickness coefficient and a_1 is the chip thickness.

The total tool-chip contact length is approximately two times larger than the sticking area. Thus this total length can be found by corresponding multiplication from formulas (1) and (2).

Many researchers suggested experimental relationships between tool-chip contact length, chip thickness, and chip thickness coefficient [5, 7–9, 12]. In some of these works, the influence of coefficient of friction was included. The length of sticking area was found to nearly equal chip thickness a_1 [9], and total tool-chip contact length is defined as:

$$L = 2 \cdot a_1. \quad (3)$$

Numerous experiments with various materials (such as armco-iron, carbon and stainless steels, different coppers, and bronzes with different hardness) and cutting conditions conducted by Poletika [5] show that tool-chip contact length is uniquely related with chip thickness coefficient ξ and undeformed chip thickness a . For the range of $1 \leq \xi \leq 10$, this dependence is approximated by the formula [12]:

$$L = a \cdot (2.05 \cdot \xi - 0.55). \quad (4)$$

One way to determine tool-chip contact length is based on the balance moment equation about tool cutting edge [2]. The main problem of this method is that the solution depends on the correctness of assumed normal stress distribution on tool-chip interface and shear plane.

A variety of ways to find tool-chip contact length gives different results for one set of cutting conditions and tool geometry. This proves that there is still no clear analytical formula that reflects the interrelationship between the shear process in primary zone and the tool-chip contact length.

4. Suggestion of new slip-line model and determination of tool-chip contact length

Fig. 3 presents the suggested slip-line field. Based on previous experimental studies [5, 17, 18], the primary deformation zone can be simplified by central slip-line field ABD, which is composed of straight rays of β -slip lines and arcs of α -slip lines. Line AB is the initial boundary of this zone while AD is the final boundary. The said lines are inclined to the tool path at angles Φ_1 and Φ_2 , respectively. Angle Φ is the conventional shear angle. Material particles are known to be hardened intensively when passing through this zone. The stress state of work material on line AB can be probably presented by yield stress for given temperature-stress-strain rate conditions of deformation.

On the final boundary AD of primary deformation zone, material hardening is saturated and the chip can be considered as ideal plastic body considering the hardening factor. The saturation of hardening in cutting has been proved by microhardness tests of quick-stop chip microsections [5]. Material hardness is not changed after deformation in primary shear zone (area ABD, Fig. 3), resulting in ultimate hardening of chip material as a result of deformation. The presence of extreme hardness itself has been also shown by Rozenberg and Rozenberg [12].

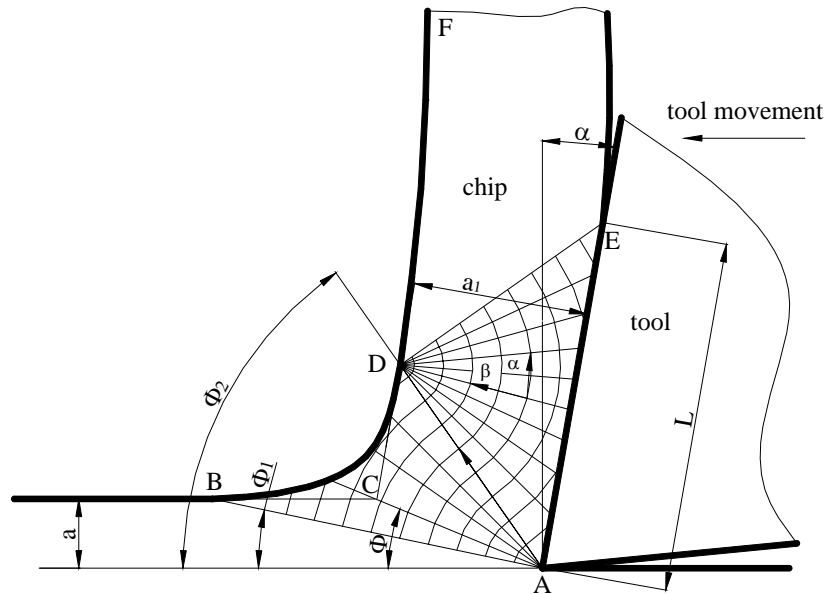


Fig. 3. Slip-line field for cutting by tool with whole rake face.

According to the theory of plasticity [17], slip-lines must intersect free transition machined/inward chip surface BD at a 45° angle. Therefore, in reality, α - and β -slip lines in the field ABD slightly differ from straight lines and arcs of circle, respectively. However, the suggested simplification is acceptable since the deflection of corresponding slip lines from straight lines and from arcs of circle is not sufficient [18].

A second central slip-line field ADE is found inside the chip body. Line DE is the final boundary of plastic zone in the chip. At point E, which is the end of tool-chip contact, it is obvious that shear stress on the tool-chip contact is zero. Thus, according to the boundary conditions for slip lines [19], line DE passes the tool rake face at a 45° angle. At the same time as the first approximation, it can be assumed that chip surface close to point D is parallel to the rake face. Since chip surface is free from the stress, line DE passes this surface at the same 45° angle according to the boundary conditions for stress. The line DE therefore is straight, passing the tool rake face and inward chip surface at the same 45° angle.

Using experimental data [14–16] discussed above, it can be assumed that in general, there is no shear stress on the tool-chip interface at tool edge (point A, Fig. 3). According to the theory of

plasticity [19], this means that angle Φ_2 in the suggested slip-line solution is constant and equal to $45^\circ + \alpha$, in other words $\angle DAE = \angle DEA = 45^\circ$, and so ADE forms an isosceles triangle.

Finally, from the suggested geometry of slip-line field, the tool-chip contact length, which is equal to line AE, is expressed by shear angle Φ and undeformed chip thickness a as:

$$L = \frac{2 \cdot a \cdot \cos(\alpha - \Phi)}{\sin \Phi}. \quad (5)$$

5. Comparative analysis of given solution for tool-chip contact length with other experimental and theoretical formulas

One of the most widespread methods to experimentally determine shear angle Φ is based on measurement of chip thickness a_1 . From the scheme presented in Fig. 3, it can be geometrically obtained that:

$$\Phi = \arctan\left(\frac{\cos \alpha}{\xi - \sin \alpha}\right). \quad (6)$$

For convenience of comparative analysis, it is better to rewrite all formulas (1) to (5) for tool-chip contact length, given above, in relative units L/a , which will be referred to as “relative tool-chip contact length”. Also, the relative tool-chip contact length is presented depending on value of chip thickness coefficient ξ .

Substituting (6) into (5) and doing some simple mathematical transformations, (5) can be reduced to a very short form:

$$\frac{L}{a} = 2 \cdot \xi. \quad (7)$$

In the same way, Lee and Shaffer’s [1] solution for total relative tool-chip contact length can be presented in the form:

$$\frac{L}{a} = \frac{\sqrt{2}}{\sin\left[\arctan\left(\frac{\cos \alpha}{\xi - \sin \alpha}\right)\right] \cdot \sin\left(\frac{\pi}{4} + \arctan\left(\frac{\cos \alpha}{\xi - \sin \alpha}\right) - \alpha\right)}, \quad (8)$$

and formula (2) by Abuladze [6] for relative tool-chip contact length comes to:

$$\frac{L}{a} = 2 \cdot (\xi(1 - \tan \alpha) + \sec \alpha). \quad (9)$$

Again, concerning relative tool-chip contact length, experimental formula (3) can be expressed by chip thickness coefficient ξ as:

$$\frac{L}{a} = 2 \cdot \xi, \quad (10)$$

and experimental dependence (4) is simply rewritten as:

$$\frac{L}{a} = (2.05 \cdot \xi - 0.55). \quad (11)$$

Comparing now the theoretical formula (7) with the experimental (10), their perfect correspondence can be easily seen.

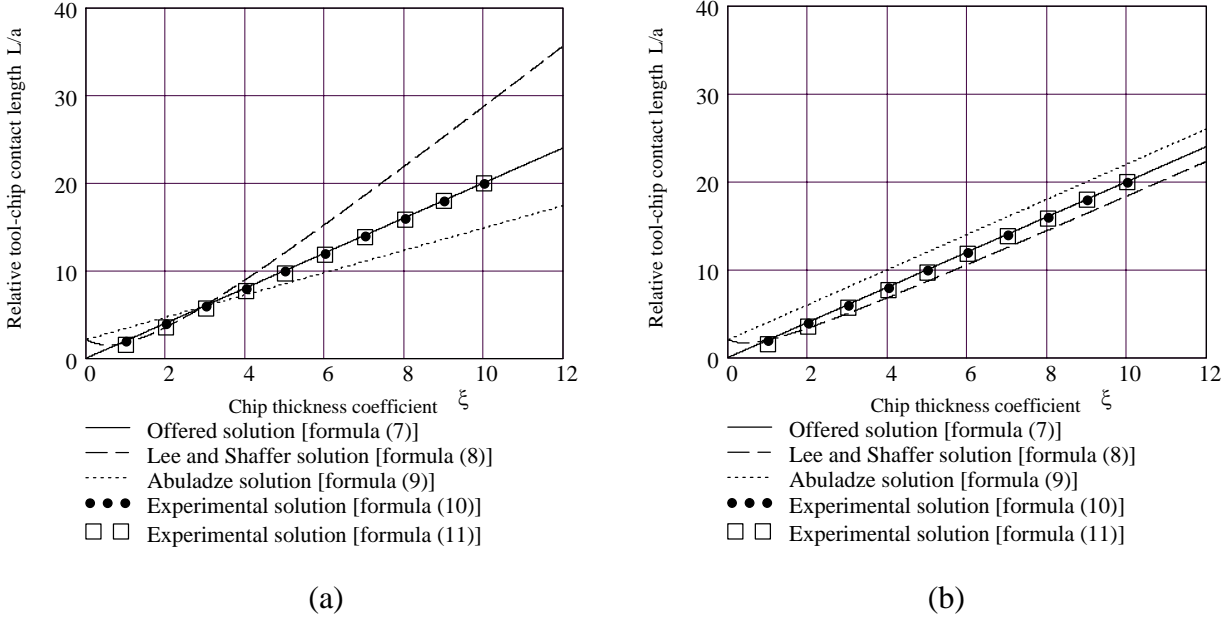


Fig. 4. Graphical comparison of different solutions for tool-chip contact length.

[(a) rake angle $\alpha = 20^\circ$, (b) rake angle $\alpha = 0^\circ$]

Fig. 4 illustrates a graphical comparison of theoretical formulas (7), (8), (9) and experimental formulas (10) and (11). The figure shows that the relative tool-chip contact length in formulas of Lee and Shaffer (8) and Abuladze (9) depends not only on chip thickness coefficient ξ but also on rake angle α . As a result, formulas (8) and (9) lead to large errors, especially for rather large rake angles. For example, the solutions of Lee and Shaffer and Abuladze shown in Fig. 4(a) are for rake angle 20° . It can be seen that for increasing value of chip thickness coefficient ξ , the error increases. If rake angle would decrease, this error would also decrease but it would still be significant as shown in Fig. 4(b), in which rake angle is 0° .

The other reason for the inaccuracy of the solutions of Lee and Shaffer and Abuladze is that they did not correspond to the mechanical sense of graphs presented in Fig. 4. It is obvious that zero chip thickness coefficient ξ means “no cutting,” and relative tool-chip contact length must be also zero. But as seen from formulas (8) and (9), this situation is impossible. The same can be said about experimental formula (11). In this case, incorrectness can be explained by unsuccessful approximation of experimental data. However, as seen in Fig. 4, the offered formula (7) by new slip-line model shows very good correspondence with the experimental results by formula (10) and (11).

6. Experimental verification

Experiments verifying the suggested method to determine chip-tool contact length were executed in CNC turning machine. Radial slots were made on cylindrical specimens to produce discs with width of 2.0–2.5 mm. These discs were machined in radial feed by a wide tool to realize the scheme of orthogonal cutting. Four different materials were used for the experiments, namely, aluminum alloy A6061, copper, carbon steel SM45C, and stainless steel STS304. Table 1 lists tool geometry and cutting conditions for each material.

The length of contact track on tool rake face was measured using tool microscope after every cutting, which was experimental tool-chip contact length. Chip thickness was also measured using a micrometer. Experimental values of relative tool-chip contact length L/a and chip thickness coefficient ξ were calculated as a result of these measurements and were then plotted (Fig. 5). The theoretical line according to formula (7) was presented in the figure for comparison with experimental data.

Table 1. Conditions of the experiment.

Work material	Cutting velocity (m/min)	Undeformed chip thickness (mm)	Tool rake angle (°)	Tool clearance angle (°)
A6061	200 - 1000	0, 0.1, 0.15, 0.2, 0.25	-5, 0, 5, 10, 20	5
Copper	200 - 800	0, 0.1, 0.15, 0.2, 0.25	-5, 0, 5, 10, 20	5
SM45C	80 - 300	0, 0.1, 0.15, 0.2, 0.25	-5, 0, 5, 10, 20	5
STS304	80 - 140	0, 0.1, 0.15, 0.2	-5, 0, 5, 10, 20	5

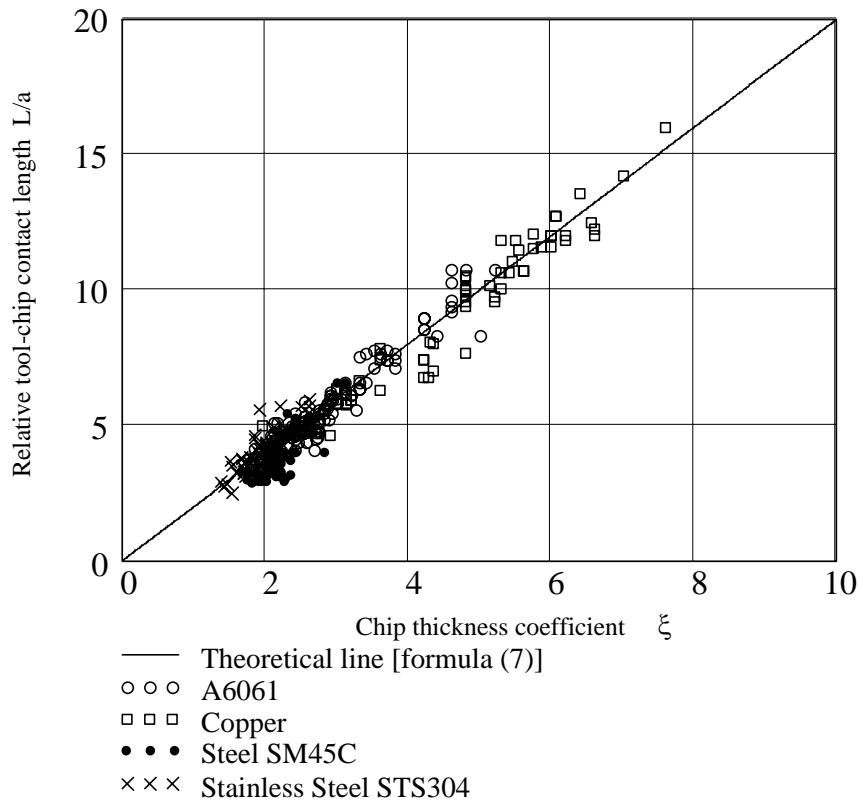


Fig. 5. Comparison of theoretical formula (7) with experimental data for different materials

Considering errors of measurements and other random experimental factors, the theoretical solution for tool-chip contact length presented in this study corresponds well for all tested materials in a wide range of cutting conditions and tool geometry.

7. Conclusion

From the given comparative analysis and this study's experimental verification, the suggested solution for tool-chip contact length was most accurate among existing analytical ways to predict this parameter. In particular, there is an exact correspondence between theoretical (7) and experimental (10) formulas for relative contact length and the same good correspondence with this study's experimental data. This remarkable coincidence proved that contours of the slip-line solution were really true for all variety of materials, cutting conditions, and rake angles for tools with whole rake face.

This study's results would be very useful for subsequent research of processes occurring on tool-chip interface, especially for determination of analytical stress distribution in that region. This study could also be helpful for analysis of tool strength, temperature phenomena, and wear problem.

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