

# Determination of stress state in chip formation zone by central slip-line field

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## ABSTRACT

Stress state of chip formation zone is one of the main problems in metal cutting mechanics. In two-dimensional case this process is usually considered as consistent shears of work material along single of several shear surfaces, separating chip from workpiece. These shear planes are assumed to be trajectories of maximum shear stress forming corresponding slip-line field. This paper suggests new approach to the constriction of slip-line field, which implies uniform compression in chip formation zone. On the base of given model it has been found that imaginary shear line in orthogonal cutting is close to the trajectory of maximum normal stress and the problem about its determination have been considered. It has been shown that there is a second central slip-line field inside chip, which corresponds well to experimental data about stress distribution on tool rake face and tool-chip contact length. The suggested model could be useful in solution of various problems of machining.

**Key Words:** Chip formation, Stress state, Slip-line solution, Stress distribution, Tool-chip contact length, Cutting forces

## Nomenclature

$k_s$  = value of plasticity corresponding to yield strength  
 $k_f$  = value of plasticity corresponding to fracture strength  
 $\sigma_0, n$  = parameters of hardening  
 $a$  = undeformed chip thickness  
 $a_1$  = chip thickness  
 $\xi = a_1/a$  = chip thickness coefficient  
 $\alpha$  = tool rake angle  
 $\Phi$  = imaginary shear angle  
 $N$  = normal force on tool rake face  
 $F$  = friction force on tool rake face

$L$  = tool-chip contact length  
 $\sigma$  = average normal stress  
 $\sigma_n(x), \tau_n(x)$  = normal and shear stress distribution on tool rake face  
 $\theta(x)$  = angle between tangent to  $\alpha$ -slip line at current point M on chip-tool interface and X-axis  
 $S_f$  = true fracture strength of work material

## 1. Introduction

Every time we consider the problems in machining, the question about stress state in chip formation zone is arisen before us. The importance of this problem follows from that stress state of work material is interrelated with

boundary conditions for stress. First of all it concerns stress distribution on tool faces, which influences on temperature fields inside tool and workpiece, defines cutting forces and tool strength, wear intensity and other engineering problems. In the case of two-dimensional cutting by tool with unrestricted rake face, it is general assumption that chip formation occurs as consistent shears of work material along single [1, 2] of several shear surfaces [3, 4], separating chip from workpiece. These shear lines in two-dimensional consideration are

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assumed to be trajectories of maximum shear stress forming corresponding slip-line field. However, these solutions concern only consideration of deformation in primary zone of chip formation. The phenomenon of chip curl suggests an idea that there is a second slip-line field inside chip [5, 6]. However all known solutions are based on the assumption about uniform stress distribution on tool-chip interface, which is not really true. The model suggested in this paper implies complex slip-line solution, which gives general law of stress state in chip formation, consistent with experimental data.

## 2. Suggested slip-line field

The suggested slip-line field is presented in Fig.1.

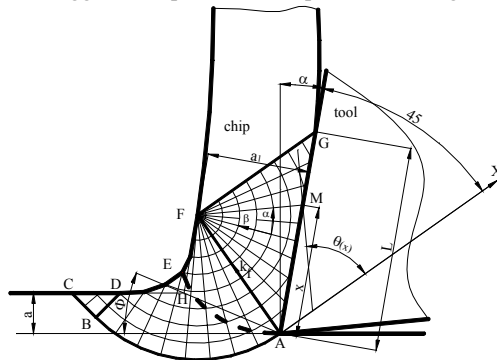


Fig.1. Slip-line solution for orthogonal cutting by tool with unrestricted rake face.

Primary deformation zone is composed on central slip-line field FABDEF and triangular field BCD. Central slip-line field FABDEF includes mutually orthogonal family of straight  $\beta$ -slip lines and arcs of  $\alpha$ -slip lines. Since chip surface DEF is free from stress, then following to the boundary conditions for slip-lines, this surface DEF is such that all rays of  $\beta$ -slip lines pass it at  $45^\circ$ . Line ABC is the initial boundary of primary deformation zone and AF is final one. Point H is imaginary intersection point of surface to be machined and free chip surface. It is assumed that tangent to the chip surface at point F is parallel to the tool rake face. Since line AF is assumed to be straight and makes an angle  $45^\circ$  with free chip surface according to the boundary conditions for slip-lines, then from the geometry of slip-line field it follows that  $\angle FAG = 45^\circ$ .

There is a second central slip-line field AFG inside chip body. Line GF is final boundary of plastic zone in the chip. Shear stress on tool-chip contact is zero at the end point G of tool-chip contact. Thus according to the

boundary conditions for slip lines, line GF passes tool rake face at angle  $45^\circ$ . Since chip surface is free from stress then line GF passes this surface at the same angle  $45^\circ$  according to the boundary conditions for stress. Hence angle AGF is equal to  $45^\circ$  what makes triangular AFG isosceles.

The suggested slip-line field gives more real form of chip formation zone than other models. Metallographic analysis and microhardness tests [7] and cutting of photoelastic materials [8] prove the similar view of initial and final boundaries of primary deformation zone. It was particularly shown that part of curvilinear initial boundary of primary zone passes under the line of tool path as in suggested solution.

## 3. Stress state of chip formation zone

Stress state of work material on the line ABC can be presented by shear yield strength  $k_s$  for given temperature-stress-strain rate conditions of deformation. As material particle moves towards the chip body, the value of plasticity  $k_s$  is changed because of hardening effect and can be found from strain-hardening characteristics  $\sigma_0$  and  $n$  considering temperature-velocity factor [9].

On the final boundary AF of primary deformation zone the material hardening is saturated and chip can be considered as ideal plastic body. In this case the value of plasticity  $k_f$  includes ultimate hardening factor. The saturation of hardening in cutting was proved by microhardness tests of quick-stop chip microsections [7] and using the method of multiple indentations [10].

Since work material is compressed in front of the tool, it follows from suggested construction of slip-line field that whole deformation area ABCDEFGA of chip formation is the zone of uniform compression. In the primary zone ABCDEFA value of plasticity is changed as result of deformation hardening. In the zone AFGA the behavior of work material is considered as ideal plastic body under the compression  $\sigma = -k_f$  with value of plasticity  $k_f$ .

The principle stresses act at  $45^\circ$  to every slip line in the field at every point, which form trajectories of maximum normal stress. Dotted line AE in Fig.1 presents this trajectory from tool cutting edge. It is easy to see that the path of this line is close to the point H that defines the location of conventional "shear" line. Thus

from suggested slip-line model it follows that conventional shear line is not a shear or slip line but trajectory of maximum normal stress. From the general relations of theory of plasticity it can be found that principle stress  $\sigma_2 = -2k'_s$  acts on this line.  $k'_s$  implies value of plasticity which considers current strain-strain rate-temperature state of work material in this line.

#### 4. Stress distribution on tool rake face

The most practically important problem in stress state of chip formation zone is contact loads on tool-chip interface. From the geometry it can be derived that the angle  $\theta(x)$  (see Fig.1) is changed for given distance  $x$  from tool edge as:

$$\theta(x) = \arctan \frac{x}{L-x} \quad (1)$$

Since chip is compressed with  $\sigma = -k_f$ , then considering obtained angle  $\theta(x)$ , the law of normal and shear stress distribution on tool rake face can be found from the theory of plasticity [11]:

$$\begin{aligned} \sigma_n(x) &= k_f (1 + \cos(2\theta(x))) \\ \tau_n(x) &= k_f \sin(2\theta(x)) \end{aligned} \quad (2)$$

The general theoretical view of stress distribution is presented in Fig.2 (b) according to formulas (2). This graph corresponds well with experimental stress distribution found by different researchers [12, 13]. Fig.2 shows the example of this correspondence according to [12].

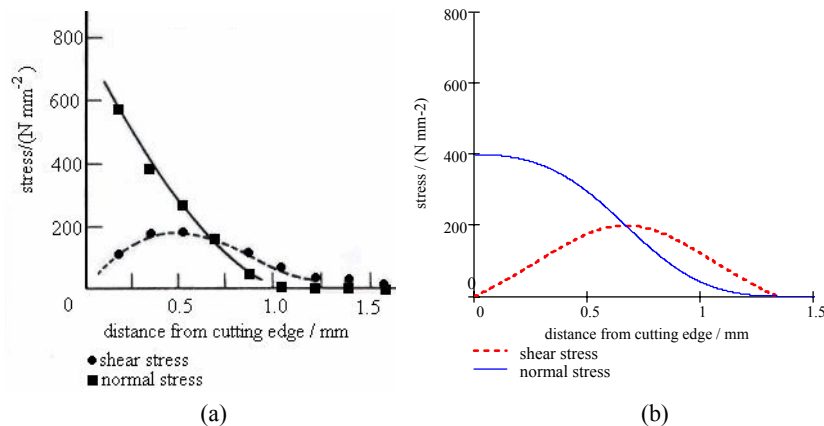


Fig.2. Experimental (a) (after [12]) and theoretical (b) (using given slip-line solution) stress distributions on rake face

#### 5. Tool-chip contact length

From geometry of suggested slip-line solution (see Fig.1) it can be obtained that tool-chip contact length is defined by the formula:

$$L = \frac{2 \cdot a \cdot \cos(\alpha - \Phi)}{\sin \Phi} \quad (3)$$

Considering chip thickness coefficient  $\xi$  this formula can be rewritten in the form [14]:

$$\frac{L}{a} = 2 \cdot \xi \quad (4)$$

It was proved in [14] that (4) corresponds well to general experimental formula in [10]. Fig. 3 gives comparison of formula (4) with experimental data for 4 different materials.

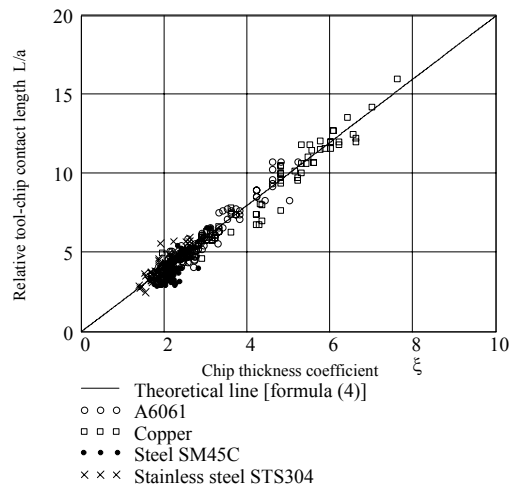


Fig. 3. Comparison of formula (4) with experimental data for different materials.

Good correspondence of predicted and experimental data

proves high accuracy of suggested formula and that contours of suggested slip-line solution are really true for wide range of cutting conditions.

## 6. Determination of forces on chip-tool interface

Integrating normal  $\sigma_n(x)$  and shear  $\tau_n(x)$  stresses on contact length  $L$  gives formulas for normal and friction forces on tool rake face:

$$N = \int_0^L \sigma_n(x) dx = L \cdot k_f, \quad (5)$$

$$F = \int_0^L \tau_n(x) dx = L \cdot k_f \left( \frac{\pi}{2} - 1 \right). \quad (6)$$

M.F. Poletika [7] found experimentally that for 22 different materials the value of average shear stress is independent on cutting conditions and tool geometry and defines by formula:

$$\tau_{n_{aver}} = const = 0.28 \cdot S_f. \quad (7)$$

From the analysis of (6) and (7) it can be concluded that value of plasticity  $k_f$  of chip material is constant and defined as:

$$k_f = const = \frac{S_f}{2}, \quad (8)$$

which is Tresca criterion of plasticity

Thus, according to Poletika's experimental data, stress state of chip corresponds to extreme stress state of material at fracture moment in tensile test, and value of plasticity  $k_f$  defines by Tresca criterion of plasticity.

## 7. Conclusion and future work

Slip-line solution suggested in this paper makes possible to give answer to various problems of metal cutting mechanics. Using this model the uniform compression stress state of chip formation zone has been found and stress distribution on tool-chip interface analytically determined. Present experimental data give good evidence for suggested model.

Further experimental research must be done to verify the accuracy of cutting force prediction and exact value of plasticity  $k_f$  must be checked. One of the questions is the analytical determination of imaginary shear angle  $\Phi$ , which is one of the main parameter in proposed solution. The analytical way to find this factor must be found.

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